$1\,dB\,cm^{-1}$, which encourages the belief that further process improvement may reduce average losses to below this value.

Discussion: We believe that the reason that guiding in Langmuir films has not previously been reported is that sufficiently thick films of good optical quality are difficult to produce. Even with our carefully optimised process we observed that, although the initial 150 to 200 were relatively defect free, the final layers were damaged by pin holes and cracking. A typical defect structure is illustrated in Fig. 2*b*. We have studied the relative scattering of the TE_0 and TM_0 modes by means of the mode lines from a prism coupler and have concluded that the increased defect density in the final layers accounts for the generally high attenuation for both TE and TM modes. However, the TM_0 mode is better confined than the TE_0 mode, interacts less with the defective region and should therefore exhibit lower losses. In our experiments, the losses are often larger for the TM_0 mode; this may be explained by the film molecules being tilted slightly with respect to the film plane. In this situation, the TE plane wave is relatively little affected, while the TM mode (in which the two electrical field components interact with both n_e and n_0) is more strongly influenced, and enhanced scattering results.

Conclusion: We have demonstrated that Langmuir films of fatty-acid salts may be deposited, by a carefully optimised process, in layers which are sufficiently thick to form optical waveguides. The thickness and refractive index, and thus the propagation constant, of such waveguides may be precisely selected. The average attenuation loss in these guides is relatively low ($\simeq 5\,dB\,cm$) and there is evidence that the intrinsic loss is even lower ($1\,dB\,cm^{-1}$) for single-mode-order structures.

Clearly, the achievement of precisely-predictable-propagation-velocity waveguides has immediate application to several device concepts, such as planar couplers. The additional facility of refractive-index and anisotropy selection by CH chain length, metal–ion inclusion and possibly by bonding aromatic side chains onto the molecule extend the applications further, e.g. fibre–film couplers. The incorporation of dyes into Langmuir films has been demonstrated.[4] This, and the metal–ion inclusions, suggest the possibility of planar laser structures using suitable Langmuir-film materials as the inert matrix.

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References

1 BLODGETT, K. B., and LANGMUIR, I.: 'Built-up films of barium stearate and their optical properties', *Phys. Rev.*, 1973, **51**, pp. 964–982
2 MANN, B., and KUHN, H.: 'Tunnelling through fatty acid salt monolayers', *J. Appl. Phys.*, 1971, **42**, pp. 4398–4405
3 NATHOO, M. H.: 'Preparation of Langmuir films for electrical studies', *Thin Solid Films*, 1973, **16**, pp. 215–226
4 WOLF, E. (ED): 'Progress in optics—Vol. XII' (North-Holland/American Elsevier, 1974)
5 BUCHER, H. *et al.*: 'The detection of the separation of monolayer dyestuff–arachidic acid films by the energy transfer method', *Z. Phys. Chem., New Series*, 1969, **65**, pp. 152–169
6 TIEN, P. K., ULRICH, R., and MARTIN, R. J.: 'Modes of propagating light waves in thin deposited semiconductor films', *Appl. Phys. Lett.*, 1969, **14**, pp. 291–294
7 PITT, C. W., GFELLER, F. R., and STEVENS, R. J.: 'Sputtered thin films for integrated optical components', *Thin Solid Films*, 1975, **26**, pp. 25–51

STACKING TECHNIQUES FOR LINEAR CODES

Indexing term: Error-correction codes

A procedure for subspace stacking is proposed, and it is proved that the technique is equally successful for both linear codes and anticodes. It is demonstrated that families of error-correcting codes may be constructed using the proposed technique for stacking linear codes. Examples of such codes are given with rates better than the best known codes of identical Hamming distance and the same number of information digits.

Introduction: A linear (n, k, d) error-correcting code is a set of q^k n-tuples that forms a subspace V of the vector space V_n over a Galois field of q elements $GF(q)$, with a minimum Hamming distance d. The codebook (array) $|0|$ is an array consisting of n columns and q^k rows,[1] the first k columns being the information part of the array and the last $n-k$ columns the parity-check (redundant) columns. The properties of subspace arrays with a maximum Hamming distance of, say, δ have been studied[2–6], and such a subspace U of vector space U_m is called an (m, k, δ) linear anticode.[2]

The techniques for stacking linear anticodes were introduced by Farrell[2] and developed further by Farrell and Farrag.[3] These stacking techniques are basically procedures for stacking a given anticode array $|0|$ and its inverse array $|I|$ (the rows of $|I|$ are logical complements of the corresponding rows of $|0|$) in such a way that the resulting array is a new linear anticode array. The purpose of this letter is to show that the anticode array stacking techniques can be generalised to subspaces which exhibit minimum and maximum distance properties; such a subspace V of V_n over $GF(q)$ is denoted by $(n_v, k_v, d_v, \delta_v)$ to indicate that V has a minimum Hamming distance d_v and maximum Hamming distance δ_v, i.e. the distance d between any two vectors in V lies between the limits $d_v \leqslant d \leqslant \delta_v$.

General stacking technique: For any two given subspaces V and U, there exists a subspace Z which results from stacking U on V. The rules of stacking and the parameters of the resulting subspace Z are given by the following theorem.

Theorem: If each of the zero elements and the nonzero elements of an $(n_v, k_v, d_v, \delta_v)$ subspace array $|0_v|$ are replaced by an $(n_u, k_u, d_u, \delta_u)$ subspace array $|0_u|$ and its inverse $|I_u|$, respectively, the resulting $(n_z, k_z, d_z, \delta_z)$ subspace array $|0_z|$ has the following parameters:

$$n_z = n_v n_u$$

$$k_z = k_v + k_u$$

d_z = smallest of the following three expressions:

(1) $n_v d_u$

(2) $n_u d_v$

(3) $n_v \delta_u + n_u \delta_v - 2\delta_v \delta_u$

δ_z = largest of the following two expressions:

(1) $n_v \delta_u$

(2) $n_u \delta_v$

Proof: Let v be a vector of weight w in the $(n_v, k_v, d_v, \delta_v)$ subspace array $|0_v|$. If each of the zero elements and each of the nonzero elements of v are replaced by the array $|0_u|$ and its inverse $|I_u|$, respectively, the resulting array has $n_v n_u$ columns and q^{k_u} rows. The minimum and maximum weight of these q^{k_u} row vectors may be determined as follows. Let the ith row vector be z:

(1) If $w = 0$, z consists of n_v, n_u-tuple vectors, such as u (vector u is the ith row vector of $|0_u|$). Since the minimum and maximum weights of the subspace U, excluding the all-zero row vector, are d_u and δ_u, respectively, the lowest possible weight of z for $w = 0$, $w_{z(min)}|_{w=0}$, excluding the all-zero row vector, is therefore $n_v d_u$, and the largest possible weight of z, $w_{z(max)}|_{w=0}$, is therefore $n_v \delta_u$.

(2) If $d_v \leqslant w \leqslant n_v/2$, z consists of w vector pairs u and their logical complements $\bar{u}$, $\{u, \bar{u}\}$, and $n_v - 2w$ of the vectors u. Moreover, the weight of the vector pairs $\{u, \bar{u}\}$ is n_u, and the lowest and largest possible weights of u are zero and δ_u, respectively. The values of $w_{z(min)}|_w$, and $w_{z(max)}|_w$, $w = d_v + p$, $p = 0, 1, 2, \ldots, n_v/2 - d_v$, are calculated and tabulated in Table 1, which shows that $w_{z(min)}|_{d_v+p}$ increases with p and $w_{z(max)}|_{d_v+p}$ decreases with p, and therefore

$$w_{z(min)}|_{d_v \leqslant w \leqslant n_v/2} = d_v n_u$$

$$w_{z(max)}|_{d_v \leqslant w \leqslant n_v/2} \leqslant n_v \delta_u$$

(3) If $n_v/2 \leqslant w \leqslant \delta_v$, z consists of $n_v - w$ of the vector pairs $\{u, \bar{u}\}$ and $2w - n_v$ of the vectors $\bar{u}$. The lowest and largest possible weights of $\bar{u}$ are $n_u - \delta_u$ and n_u, respectively. The values of $w_{z(min)}|_{n_v/2+j}$, and $w_{z(max)}|_{n_v/2+j}$,

$$j = 0, 1, 2, \ldots, \delta_v - n_v/2,$$

are tabulated in Table 1, from which it may be deduced that

$$w_{z(min)}|_{n_v/2 \leqslant w \leqslant \delta_v} = n_v \delta_u + n_u \delta_v - 2\delta_v \delta_u$$

$$w_{z(max)}|_{n_v/2 \leqslant w \leqslant \delta_v} = n_u \delta_v$$

This completes the proof.

Consider the subspace Z of the above theorem. δ_z was found to be independent of d_v and d_u, so this implies that any two given anticodes (m_v, k_v, δ_v) and (m_u, k_u, δ_u) may be stacked to generate a new (m_z, k_z, δ_z) anticode Z. However, to generate a linear code by the stacking technique, the minimum and maximum distance of two subspaces must be known. Such subspaces exist with length $n = 2^k - 1$ and $d = \delta = (n+1)/2$, known as m-sequence codes (or equidistance codes). A large family of stacked codes may therefore be generated by stacking m-sequence codes on each other. For example, consider the stacking of the $(3, 2, 2, 2)$ m-sequence code on itself: $n_v = n_u = 3$, $k_v = k_u = 2$ and $d_v = \delta_v = d_u = \delta_u = 2$.

$$|0_v| \equiv |0_u| \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$|0_z| \rightarrow \begin{bmatrix} 0_u & 0_u & 0_u \\ I_u & 0_u & I_u \\ 0_u & I_u & I_u \\ I_u & I_u & 0_u \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \rightarrow \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \\ z_{10} \\ z_{11} \\ z_{12} \\ z_{13} \\ z_{14} \\ z_{15} \\ z_{16} \end{bmatrix}$$

z_7, z_{14}, z_{16} and z_9 form basis vectors for the subspace array $|0_z|$, the generator matrix $[G_z]$ of the resulting $(9, 4, 4)$ stacked code is therefore given by

$$n_z = 9, \ k_z = 4, \quad d_z = 4$$

$$[G_z] \rightarrow \begin{bmatrix} z_7 \\ z_{14} \\ z_{16} \\ z_9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

In general, if a subspace U of $GF(q)$ stacked on a subspace V of $GF(q)$, the generator matrix $[G_z]$ of the resulting subspace Z may be determined as follows:

Let the generator matrix $[G_u]$ of the $(n_u, k_u, d_u, \delta_u)$ subspace U given by

$$[G_u] \rightarrow \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{k_u} \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n_u} \\ a_{21} & a_{22} & \ldots & a_{2n_u} \\ \vdots & & & \vdots \\ a_{k_u 1} & a_{k_u 2} & \ldots & a_{k_u n_u} \end{bmatrix}$$

and that of $(n_v, k_v, d_v, \delta_v)$ subspace V be

$$[G_v] \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{k_v} \end{bmatrix} \rightarrow \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1n_v} \\ b_{21} & b_{22} & \ldots & b_{2n_v} \\ \vdots & & & \vdots \\ b_{k_v 1} & b_{k_v 2} & \ldots & b_{k_v n_v} \end{bmatrix}$$

where a_{ij} and b_{ij} are elements of $GF(q)$. A basis vector set of Z may be simply described by two vector sets; the first consists of k_u n_z-tuples resulting from stacking the vectors $u_1, u_2, \ldots, u_{k_u}$ on the all-zero vector of V; and the second set consists of k_v n_z-tuples resulting from stacking the all zero vector of U on the vectors $v_1, v_2, \ldots, v_{k_v}$ of V. The $k_u + k_v$ rows of the resulting matrix are linearly independent and the $[G_z]$ matrix therefore has the form

$$[G_z] \rightarrow \underbrace{\left[\begin{array}{ccc|ccc|c|ccc} & u_1 & & & u_1 & & \ldots & & u_1 & \\ & u_2 & & & u_2 & & \ldots & & u_2 & \\ & \vdots & & & \vdots & & & & \vdots & \\ & u_{k_u} & & & u_{k_u} & & \ldots & & u_{k_u} & \\ b_{11} & b_{11} \ldots & b_{11} & b_{12} & b_{12} \ldots & b_{12} & \ldots & b_{1n_v} & b_{1n_v} \ldots & b_{1n_v} \\ b_{21} & b_{21} \ldots & b_{21} & b_{22} & b_{22} \ldots & b_{22} & \ldots & b_{2n_v} & b_{2n_v} \ldots & b_{2n_v} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & \vdots & \vdots \\ b_{k_v 1} & b_{k_v 1} \ldots & b_{k_v 1} & b_{k_v 2} & b_{k_v 2} \ldots & b_{k_v 2} & \ldots & b_{k_v n_v} & b_{k_v n_v} \ldots & b_{k_v n_v} \end{array}\right]}_{n_v n_u} \ k_v + k_u$$

Table 1

Weight of vector v (w)	Corresponding minimum weight of z for a given weight of v $(w_{z(min)}\|_w)$	Corresponding maximum weight of z for a given weight of v $(w_{z(max)}\|_w)$
0	$\boxed{n_v\, d_u}$	$\boxed{n_v\, \delta_u}$
d_v	$\boxed{n_u\, d_v}$	$n_v\,\delta_u - d_v(2\delta_u - n_u)$
$\vdots$	$\vdots$	$\vdots$
d_v+p	$n_u(d_v+p)$	$n_v\,\delta_u - (d_v+p)\,(2\delta_u - n_u)$
$\vdots$	$\vdots$	$\vdots$
$n_v/2$	$n_u(n_v/2)$	$n_u(n_v/2)$
$\vdots$	$\vdots$	$\vdots$
$n_v/2+j$	$n_u(n_v/2)-j(2\delta_u-n_u)$	$n_u(n_v/2)+j$
$\vdots$	$\vdots$	$\vdots$
δ_v	$n_v\,\delta_u + n_u\,\delta_v - 2\delta_v\,\delta_v$	$\boxed{n_u\,\delta_v}$

Table 2

Stacked codes $(n_v, k_v, d_v, \delta_v)$ and $(n_u, k_u, d_u, \delta_u)$	New code Z (n_z, k_z, d_z)	d_L, d_{up}
(2, 1, 1, 1), (60, 8, 28, 32)	(120, 9, 56)	
	(119, 9, 55)	54, 57
(2, 1, 1, 1), (59, 8, 27, 32)	(118, 9, 54)	
	(117, 9, 53)	52, 56
(31, 5, 16, 16), (3, 2, 2, 2)	(93, 7, 46)	
	(92, 7, 45)*	44, 45

* This code can also be found by the Solomon and Stiffler[8] puncturing technique
d_L = Helgert and Stinaff[7] lower bounds on minimum Hamming distance
d_{up} = Helgert and Stinaff[7] upper bounds on minimum Hamming distance

This form of the matrix does not directly generate the systematic form of the code; however, the reduced echelon form of $[G_z]$ may be obtained by simple elementary row operations.[1]

A few examples of new codes found by code stacking are given in Table 2.

Comments: The weight distribution of some classes of linear codes are known[9, 10] so d and δ for such codes can be determined and thus the appropriate families of stacked codes can be constructed. It is useful to note that if code U is stacked on code V, where both V and U are cyclic codes, the resulting code Z, by definition, is a quasicyclic code. The useful properties of quasicyclic codes may therefore be extended to construct encoding and decoding algorithms for codes derived by the stacking procedure.

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References

1 PETERSON, W. W., and WELDON, E. J., JUN.: 'Error-correcting codes', (MIT Press, 1972, 2nd edn.)
2 FARRELL, P. G.: 'Linear binary anticodes', *Electron. Lett.*, 1970, **6**, pp. 419–421
3 FARRELL, P. G., and FARRAG, A.: 'Further properties of linear binary anticodes', *ibid.*, 1974, **10**,
4 MARK, G. K., and TRACY, J. H.: 'Maximum distance linear codes', *IEEE Trans.*, 1971, **IT–17**, p. 637
5 REDDY, S. M.: 'On block codes with specified maximum distance', *ibid.*, 1972, **IT–18**, pp. 823–824
6 HASHIM, A. A.: 'Maximum distance bounds for linear anticodes', *Proc. IEE*, 1976, **123**, (3), pp. 189–190
7 HELGERT, H. J., and STINAFF, R. D.: 'Minimum-distance bounds for binary linear codes', *IEEE Trans.*, 1973, **IT–19**, pp. 344–356
8 SOLOMON, G., and STIFFLER, J. J.: 'Algebraically punctured cyclic code', *Inf. & Control*, 1965, (8), pp. 170–179
9 PETERSON, W. W.: 'On the weight structure and symmetry of BCH codes', *J. Inst. Elec. Commun. Eng. Japan*, 1976, **50**, pp. 1183–1190
10 GOETHALS, J. M.: 'Analysis of weight distribution in binary cyclic codes', *IEEE Trans.*, 1966, **IT–12**, pp. 401–402

HYDROGEN SENSITIVITY OF PALLADIUM–THIN-OXIDE–SILICON SCHOTTKY BARRIERS

Indexing terms: Metal–insulator–semiconductor structures, Schottky effect, Tunnelling

It is shown that current transport through Schottky barriers formed by Pd on n-type silicon with a thin thermally grown oxide is sensitive to hydrogen in the ambient. It is shown that a transition from minority- to majority-carrier dominated current occurs with increasing hydrogen pressure.

Metal–thin-oxide–silicon Schottky barriers have been described by several authors.[1–4] The thin oxide decreases the density of surface states considerably, which otherwise would dominate the I/V characteristics of the barriers. The energy barrier between the metal and the semiconductor is therefore largely determined by the difference in their work functions. It has been shown theoretically and experimentally that, with a thin oxide layer, both minority- and majority-carrier dominated structures can be obtained.[2–4] In the first case, the current through the structure is determined by minority carriers diffusing to the interface between the semiconductor and the oxide, and their tunnelling through the oxide. In the second case, the current is dominated by thermally excited majority carriers tunnelling through the oxide ('normal Schottky current').

We have recently shown that hydrogen introduces a dipole layer at the interface between catalytically active metals like Pd and Pt and an insulator. This changes the energy barrier at the interface, i.e. the work-function difference between the metal and the semiconductor on which the insulator is grown.[6–10] Hydrogen is dissociated on the outer metal surface and dissolved in the metal in atomic form. Some of the dissolved hydrogen is adsorbed onto the metal–insulator interface, where it gives rise to a dipole layer. The description above is for temperatures and hydrogen con-

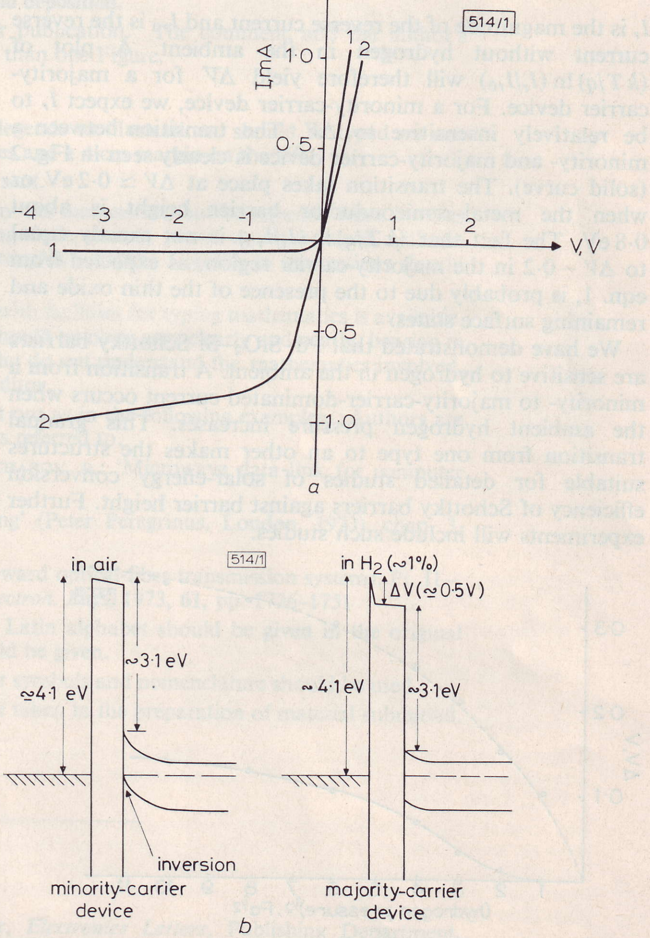

Fig. 1

(*a*) I/V curves of a Pd–SiO_2–n-type-Si Schottky barrier in air (curve 1) and in a small amount of hydrogen, 1% H_2, in air (curve 2). The temperature of the Schottky barrier was 150°C.
(*b*) Schematic band diagram of the Schottky barrier in air (left) and in hydrogen (right), respectively. In air (curve 1 in *a*), the current is dominated by minority carriers. In 1% H_2 (curve 2 in *a*), the current is dominated by majority carriers.